

Can a single gradientless light beam drag particles?

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Usually a light beam pushes a particle when the photons act upon it. This is due to that the electric-dipole particle in the paraxial beam is considered. We investigate the scattering forces in non-paraxial gradientless beams and find that the forces can drag certain particles towards the beam source. The major criterion to be carried out to get the attractive force is the strong non-paraxiality of the light beam. The cone angle denoting the non-paraxiality has been investigated to unveil its importance on achieving dragging force. We hope the attractive forces will be very useful in nanoparticle manipulation.

In spite of the well established theory, experiment and applications in particle-light interaction [1–5], there are many unknowns yet to be explored in the particular field of attracting or separating molecules or nanoparticles by gradientless light beams. In the ordinary optical tweezers, the transfer of the particles along the 3D trajectories can be achieved using the spatial light modulators [6]. In this case, the consecutive change of the computer-generated holograms for the tightly focused light beams produces the necessary effect, trapping of the particle due to the gradient of the field.

It is well known that the light beam results in repelling force on the object. It is correct within electric-dipole approximation and paraxial light beams, when the size of the particle is much less than the wavelength [2]. In this case, the force can still be manipulated to be a dragging force, provided that two oppositely directed beams are present to control the particle's position [7]. However, such two-beam configuration (requiring exactly opposite directions) is not necessary to achieve the attractive force. As shown in Ref. [8], two beams with different longitudinal wavenumbers can locally operate as a tractor beam. Another possibility to get attractive force is to use the gain media [9], but the exotic media strongly confine the possible applications of the *negative* forces. In Ref. [10] it was suggested without proof that the negative Poynting vector could be the reason of the attractive force. Though the negative energy flux density is not the reason of the particle drag towards the light source, this effect is quite possible in the non-paraxial beams.

Small particles (hence, the dipole approximation) as a limiting factor are not necessary either. The particle can be large compared to the wavelength and positioned near the center of the beam. In this letter we use a non-paraxial gradientless beam (a vector Bessel beam) to induce the dragging force. The choice is owing to the following reasons: (i) long propagation distance, where beams weakly spread and keep their intensity profile; (ii) complex polarization structures of the beam; (iii) non-paraxiality of the beams denoted by the cone angle. We need such a general description of the beam, because it

would be nice to know whether the beam should be actually complex or we can use its simplified version. Field distribution in the form of the vector Bessel beam is not unusual. It can be generated as a mode of a circular fiber. However, the experimental generation of the non-paraxial beams is still challenging.

The key idea is to demonstrate that a gradientless beam can exert a dragging force upon a particle. The beam being non-paraxial and gradientless is very important because the particle can thus be directly dragged toward the light source realizing the idea of the tractor beam. It should be noted that the Bessel beam reconstructs the field behind the particle [11]. This means that we can trap and drag several objects by a single beam.

z -propagating electromagnetic Bessel beam in vacuum is described as [10]

$$\begin{aligned}\mathbf{E} &= e^{im\varphi + i\beta z} \left(J_m(qr) c_2 \mathbf{e}_z - \frac{k_0}{q} c_1 (\mathbf{e}_z \times \mathbf{b}) + \frac{\beta}{q} c_2 \mathbf{b} \right), \\ \mathbf{H} &= e^{im\varphi + i\beta z} \left(J_m(qr) c_1 \mathbf{e}_z + \frac{\beta}{q} c_1 \mathbf{b} + \frac{k_0}{q} c_2 (\mathbf{e}_z \times \mathbf{b}) \right),\end{aligned}\quad (1)$$

where $k_0 = \omega/c$ is the wavenumber in vacuum, $q = k_0 \sin \alpha$ is the transverse wavenumber (α is the cone angle, see Fig. 1 (a)), $\beta = \sqrt{k_0^2 - q^2}$ is the longitudinal wavenumber, m is the beam order,

$$\mathbf{b} = iJ'_m(qr)\mathbf{e}_r - \frac{m}{qr}J_m(qr)\mathbf{e}_\varphi, \quad J'_m(qr) = \frac{dJ_m}{d(qr)}.$$

The typical intensity distribution $|\mathbf{E}|^2$ of the vector Bessel beam is illustrated in Fig. 1(b) and (c). The intensity is actually propagation invariant (z -independent) and azimuth invariant (φ -independent). It worth to draw attention to the coefficients c_1 and c_2 of the vector Bessel beam (1). They provide the possibility to create the appropriately polarized propagation invariant field distribution and, in particular, can be associated with TE-polarized ($c_2 = 0$) and TM-polarized ($c_1 = 0$) waves. In general, coefficients c_1 and c_2 can be complex numbers, what means that the TE and TM beams are phase

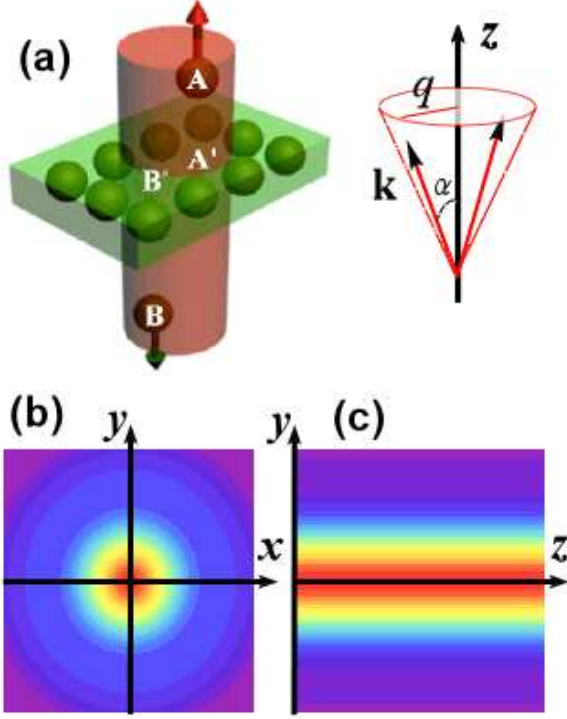


FIG. 1: (a) Light beam pushes a particle from position B' to position B while drags another particle from original position A' to new position A (it will keep dragging if the light keep shining). On the right, the gradientless Bessel beam wavevectors lying on the cone are sketched. (b) and (c) denote the cross-sections of the intensity of such a beam propagating along z-axis ($c_1 = 1$, $c_2 = i$, $q/k_0 = 0.9$, $m = 1$).

shifted. The ordinary Bessel beams follow from Eq. (1) in paraxial approximation ($q \ll k_0$), but then we lose both possibility to set large qs and generate beams with complex polarization structure. Non-paraxial Bessel beams have already demonstrated a number of exciting properties like local negative direction of the Poynting vector [10] and intensity transformation on reflection [12].

We assume that the spherical particle of the radius R is situated exactly at the center of the beam, but it is not a limitation, because the calculations can be made for the shifted particle as well. The incident vector Bessel beam is scattered by the particle according to the Mie theory. The scattered fields can be calculated, for example, using the matrix approach [13, 14], which is valid for any incident electromagnetic wave. So, the fields at the boundary of the sphere can be written as the sum of the incident and scattered fields: $\mathbf{E} = \mathbf{E}^{inc} + \mathbf{E}^{sc}$ and $\mathbf{H} = \mathbf{H}^{inc} + \mathbf{H}^{sc}$. These fields are sufficient to compute the time-averaged electromagnetic force on the particle:

$$\mathbf{F} = \int_0^\pi \int_0^{2\pi} (\hat{\mathbf{T}} \mathbf{n}) R^2 \sin \theta d\theta d\varphi, \quad (2)$$

where \mathbf{n} is the outward normal, time-averaged Maxwell's

stress tensor is

$$\hat{T} = \frac{1}{8\pi} \text{Re} \left(\mathbf{E} \otimes \mathbf{E}^* + \mathbf{H} \otimes \mathbf{H}^* - \frac{1}{2} (|\mathbf{E}|^2 + |\mathbf{H}|^2) \right). \quad (3)$$

Here $\mathbf{E} \otimes \mathbf{E}^*$ defines the dyad, which in index form is $(\mathbf{E} \otimes \mathbf{E}^*)_{ij} = E_i E_j^*$. We use the general formulae for calculations, but the scattered fields of the Rayleigh particles can be written via the electric and magnetic dipole moments [3]. We are interested only in the component F_z of the force and look for the situations when $F_z < 0$, i.e. when the particle is pulled by the beam.

When the particle is located at the beam axis, angle φ for the Bessel beam (1) is simultaneously the azimuthal angle of the particle's spherical coordinates. Therefore, the dependence of the fields on φ keeps only as an exponential $\exp(im\varphi)$ and the incident and scattered fields contain the single term in the sum over the integer azimuthal number. It is not the case for the particle shifted from the axis. Then we need to take into account also the terms with integer azimuthal numbers nonequal m .

The structure of the beam strongly defines the possibility of the attractive force. However, the intensity of the light is not the criterium of the pulling. Even identical intensity patterns (like in the insets of Fig. 2 (b) and (d)) result in the qualitatively different dependencies of the force. It is likely that the vector (polarization) structure of the beam plays the leading role in the appearance of the attractive force. It should be noted that the negative longitudinal component of the Poynting vector S_z is not responsible for the appearance of the negative force, because the large negative S_z exists for $c_2 = -i$ [10], while the force is positive in this case (Fig. 2(f)).

Strong attractive force exists for non-phase-shifted superposition of TE and TM-polarized beams (c_2 is positive real number as in Fig. 2(b) and (c)) and for $\pi/2$ -shifted beams (c_2 is imaginary number with $\text{Im}c_2 > 0$ as in Fig. 2(e)). In the latter case, the negative force is broadband, low positive peaks being caused by the quadrupole terms (by the integer polar number $l = 2$ in the scattering series). These two peaks at $\varepsilon = 8$ and $\varepsilon = 10$ exist in each subfigure and provide the positive force. The dipole term ($l = 1$) describes the features near $\varepsilon = 4$. Due to the non-paraxial regime, and/or large particle size, and/or magnetic moments the force owing to the dipole term $l = 1$ can become attractive.

Thus, the Bessel beams with $c_2 = 1$ and $c_2 = i$ are preferred, but they are quite different. In Fig. 3 we show the force acting on the particle for $c_2 = 1$ (subfigures (a) and (c)) and $c_2 = i$ (subfigures (b) and (d)). Asymmetry in the case of $c_2 = 1$ implies that, in order to induce the attractive force the permittivity should be greater than the permeability, i.e. the excited electric moments play the main part. In the symmetric case (Fig. 3(b) and (d)), both electric and magnetic inputs are of the same order and the force has a minimum at the impedance matching line $\varepsilon = \mu$. It is interesting that the region of the negative force is really great. The value of the force can be enhanced by adjusting the radius of the sphere

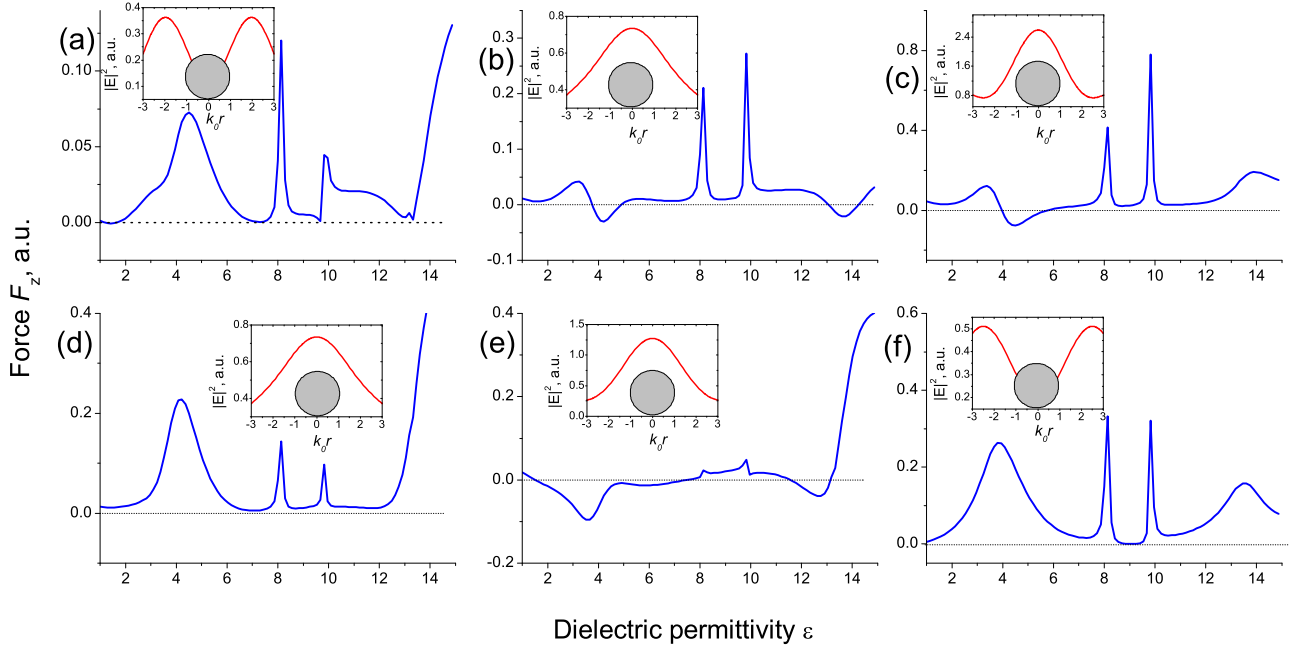


FIG. 2: Force F_z on the spherical particle ($k_0 R = 1$, $\mu = 3$) versus dielectric permittivity ϵ for (a) $c_2 = 0$, (b) $c_2 = 1$, (c) $c_2 = 2$, (d) $c_2 = -1$, (e) $c_2 = i$, and (f) $c_2 = -i$. In the insets, the field intensities are demonstrated (grey circle stands for the particle). Beam parameters are $c_1 = 1$, $q/k_0 = 0.9$, and $m = 1$.

(see Fig. 4(a)). From the figure Fig. 3(d) we can guess that the force can be attractive even for $\mu = 1$ (it is out of the range of the plot). It is actually the case as can be seen from Fig. 4(a). So, the dragging force can act even on the common glass particles, if it has an appropriate size. The maximizing of the absolute value of the negative force using the impedance matching condition substantially broadens the range of the negative force. The only strong limitation on the vector Bessel beams is their non-paraxiality. As shown in Fig. 4 (b), the variation of the parameters does not substantially reduce the cone angle $\alpha = \arcsin(q/k_0)$ (for $q = 0.9$ the angle α equals $\approx 64^\circ$). The beam should have small longitudinal wavenumber β to be able to create the attractive force.

From Fig. 4(a) we can suggest that even for the Rayleigh magnetodielectric particles the force can be less than zero. In fact, the mechanism behind the attractive force can be revealed from the theory of magnetoelectric Rayleigh particles [3]. Using the fact of the small imaginary part of the polarizability in comparison with the real part, we derive

$$\langle F_z \rangle = \frac{\beta k_0}{2} (\text{Im}(\alpha_e) |\mathbf{E}|^2 + \text{Im}(\alpha_m) |\mathbf{H}|^2) - \frac{k_0^4}{3} \text{Re}(\alpha_e) \text{Re}(\alpha_m) \text{Re}(P_z), \quad (4)$$

where $P_z = \mathbf{e}_z (\mathbf{E} \times \mathbf{H}^*)$, α_e and α_m are the electric and magnetic polarizabilities of the particle, respectively. From Eq. (4) it is evident that the negative force is feasible due to (i) *small* longitudinal wavenumber β of the light beam, (ii) existence of the *magnetic* dipole moment,

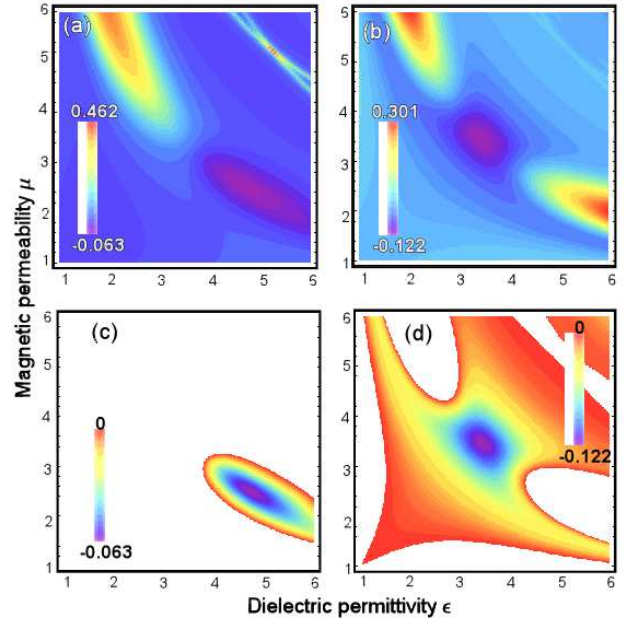


FIG. 3: Diagram of the force F_z as function of the dielectric permittivity ϵ and magnetic permeability μ for (a), (c) $c_2 = 1$ and (b), (d) $c_2 = i$. In the bottom subfigures, only negative values of the force is shown. Parameters: $k_0 R = 1$, $c_1 = 1$, $q/k_0 = 0.9$, $m = 1$.

and (iii) large *positive* Poynting vector (quantity $\text{Re}(P_z)$).

If β is not small, what is the case of the small spherical particles ($\alpha_e = \alpha_e^{(0)} + i2k_0^3 \alpha_e^{(0)2}/3$, where $\alpha_e^{(0)} = a^3(\epsilon -$

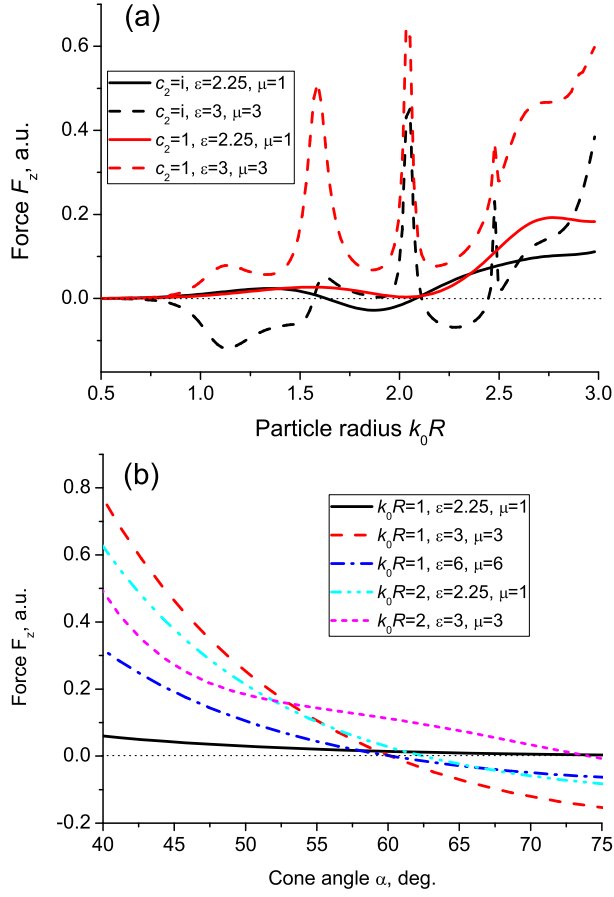


FIG. 4: Dependence of the z -component of the force on (a) the radius of the spherical particle $k_0 R$ ($q/k_0 = 0.9$) and (b) cone angle $\alpha = \arcsin(q/k_0)$ ($c_2 = i$). A multimedia file is created to show the significance of the cone angle (non-paraxiality effect) and particle's radius (size effect), in order to achieve dragging forces (see Media 1. Hyperlink: <http://www.ece.nus.edu.sg/stfpage/eleqc/force.gif>). Parameters: $c_1 = 1$, $m = 1$.

$1)/(\epsilon + 2)$, and the similar expressions for the magnetic

polarizability) in the field of the plane wave ($\beta = 1$ and $|\mathbf{E}|^2 = |\mathbf{H}|^2 = \text{Re}(P_z) = 1$) we write

$$\langle F_z \rangle = \frac{k_0^4 a^6}{3} \left(\frac{(\epsilon - 1)^2}{(\epsilon + 2)^2} + \frac{(\mu - 1)^2}{(\mu + 2)^2} - \frac{(\epsilon - 1)(\mu - 1)}{(\epsilon + 2)(\mu + 2)} \right).$$

The force F_z cannot be attractive for any dielectric permittivity and magnetic permeability under the illumination of a plane wave.

For non-magnetic small particles ($\alpha_m = 0$), the force is always pushing (it follows straightforwardly from Eq. (4)). Nevertheless, non-magnetic non-Rayleigh particles can be attracted owing to the contributions of the higher-order electrical moments.

Eq. (4) claims the Poynting vector should have large positive value. From [10] we notice that the largest value of the Poynting vector is achieved for $c_2 = i$ and this fact is in full agreement with the calculations in Fig. 2. However, we should be careful in generalization of the results for the Rayleigh particle to the non-Rayleigh ones. Indeed, the Poynting vector distributions are the same for $c_2 = 1$ and $c_2 = -1$, but the forces dramatically different in the case $k_0 R = 1$ as it is observed in Fig. 2 (b) and (d). So, Eq. (4) does not contradict the existence of the attractive force for magnetodielectric Rayleigh particles.

In conclusion, we have demonstrated the possibility of the attractive forces from a gradientless light beam. Compared to the previously reported tractor beams [8], we have the broadband negative force and do not need two opposite beams with different wavenumbers [7] or one gradient beam [6]. The proposed gradientless beam with the proper manipulation from the non-paraxiality can lead to a lasting dragging force exerted on particles which can be large or small and made of any materials.

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